NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE

No. 1301

A METHOD FOR CALCULATING THE HEAT REQUIRED FOR THE PREVENTION OF

FOG FORMATIONS ON THE INSIDE SURFACES OF

SINGLE-PANEL BULLET-RESISTING WINDSHIELDS DURING DIVING FLIGHT

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SUMMARY

An investigation was conducted to provide a means for calcu-
lating the heat required for the prevention of fog formations on
the inside surfaces of single-panel bullet-resisting windshields
during diving flight. An analysis was made to provide relation-
ships for the heat required considering the transient heating of
the windshield during diving flight. From the results of this
analysis, it is evident that for dives where the rate of descent
is high, over 5000 feet per minute, the temperature of the inside
surfaces of a single-panel bullet-resisting windshield remains
approximately constant unless there is an appreciable change of
heat input to the windshield during the dive. Thus, in designing
fog-prevention systems for bullet-resisting windshields, the design
may be carried out for steady-state conditions at the altitude from
which diving flight is initiated unless there is an appreciable
decrease of heat input through the windshield with decreasing
altitude. In this event consideration of the transient heat flow
during diving flight may be necessary in order to provide fog
protection. Consideration of the transient heat flow during
diving flight may also be desirable in order to conserve heat
energy when the heat input to the windshield increases appreciably
with decreasing altitude.

Consideration is given to the use of plane heated-air jets
directed tangentially to the inside surface of the windshield to
provide the required heat. A relationship is developed to define
the optimum depth of the jet nozzle for any given set of conditions.
It is also shown that it is desirable to maintain the cockpit air
temperature and the jot-nozzle exit air temperature as high as
practical. The application of heated-air jets to the fogging problem
is discussed for both steady-state and transient solutions of the
heat required, and a sample solution is presented for each case.

Consideration is also given to the use of electrical energy to heat the inside surface of a windshield. It is shown, considering only free convection and radiation of heat from the windshield to the cockpit, that electrical heating systems may be designed using steady-state heat-transfer considerations.

INTRODUCTION

During diving flight of military aircraft equipped with bullet-resisting windshields, the inside surfaces of the windshields frequently accumulate fog which obscures the pilot's vision and renders the striking power of the aircraft ineffective. The fog may be removed by mechanical means; however, these means in themselves tend to impair the pilot's vision. The fog may be prevented from forming by maintaining the windshield inside-surface temperature above the dew point of the adjacent cockpit air. This can be accomplished by either providing for a flow of dehydrated air over the inside of the windshield or by heating the inside surface of the windshield. Dehydration of the air could be accomplished by chemical means or by use of a refrigeration cycle. Chemicals are undesirable because constant replacement of the chemicals is necessary and refrigeration is not practical at present, in that present-day aircraft are not equipped with refrigeration equipment. Heating of the windshield could be accomplished simply as most aircraft are equipped with heated-air systems or with electrical energy which can readily be converted into heat.

Since there were no design data available which considered the basic variable involved in providing fog protection by heating the inside surfaces of the windshields, the present investigation was undertaken.

The purpose of the present investigation was to provide a means for evaluating the heat input to the inside surface of a single-panel bullet-resisting windshield which is required to maintain the temperature of the inside surface of the windshield above the dew point of the adjacent cockpit air throughout diving flight.

Equations are derived for the calculation of the heat required throughout diving flight in which the transient heating of the windshield is considered. The results of this analysis are compared with a simple steady-state solution which provides for heating the inside surface of the windshield to the highest temperature required during flight at the steady-state conditions existing prior to the dive and

neglects calculation of transient heat flow in the windshield. After
establishing relationships for the heat required, consideration is
given to the application of plane heated-air jets (hereinafter
designated as surface jets) to provide the required heat using the
relationships given in reference 1 and also to the application of
electrical energy to provide the required heat.

SYMBOLS

The following symbols are employed throughout this report:

a	thermal diffusivity of windshield material, square feet per hour
c_k	percent of full kinetic heating $\left(c_k = \dfrac{\Delta T_k 2gJc_p}{V^2} \right)$
c_p	specific heat of air at constant pressure, Btu per pound, $^\circ F$
c	specific heat of windshield material, Btu per pound, $^\circ F$
d	surface-jet-nozzle depth, feet
d_1	reference surface-jet-nozzle depth of 1/12 foot, feet
e	distance from surface-jet origin to nozzle exit $\left(\dfrac{d}{\tan \alpha} \right)$, feet
g	acceleration due to gravity, feet per second, second
h_c	coefficient of free convection and radiation of heat at the electrically heated surface of a windshield, Btu per hour, square foot, $^\circ F$
h_o	coefficient of heat transfer at external surface of windshield, Btu per hour, square foot, $^\circ F$
h_i	coefficient of heat transfer at internal surface of windshield, Btu per hour, square foot, $^\circ F$
h_j	coefficient of heat transfer at internal surface of windshield employing heated-air-surface jet, Btu per hour, square foot, $^\circ F$

H   pressure altitude, feet

J   mechanical equivalent of heat, 778 foot-pounds per Btu

k_w thermal conductivity of windshield material, Btu per
             hour, square foot, °F per foot

k_a thermal conductivity of air, Btu per hour, square foot,
             °F per foot

k_1, k_2, k_3 functions of altitude such that

$$(T_1)_{n\Delta t, o} = (k_1)_{n\Delta t} q_a + (k_2)_{n\Delta t} \psi + (k_3)_{n\Delta t}$$

L_w length of windshield, feet

L   distance from surface-jet-nozzle exit to point under
             consideration (L = x-e), feet

l width of surface-jet nozzle, feet

q_a heat flow through the inside surface of windshield at
             steady-state conditions prior to dive flight, Btu per
             hour, square foot

q_E heat generated at the inside surface of a windshield by the
             conversion of electrical energy to heat energy, Btu per
             hour, square foot

q_H change in heat flow through the inside surface of wind-
             shield at any time after dive flight is started, Btu
             per hour, square foot

q_i heat transferred from the electrically heated inside surface
             of a windshield into the cockpit, Btu per hour, square
             foot

t   time after start of dive flight, hours

T   temperature at any point in windshield, °F

T_c temperature of cockpit air, °F

T_i temperature of inside surface of windshield, °F

T_a static ambient-air temperature, °F

T_k kinetic ambient-air temperature, $^\circ F$

ΔT_k ambient-air-temperature change due to kinetic heating
                $(\Delta T_k = T_k - T_a)$, $^\circ F$

$T_{D.P.}$ dew-point temperature of cockpit air, $^\circ F$

ΔT difference between temperature of inside surface of
                windshield and dew-point temperature of the cockpit
                air $(\Delta T = T_1 - T_{D.P.})$, $^\circ F$

T_o temperature of air at surface-jet-nozzle exit, $^\circ F$

T_j maximum jet temperature at any distance x from
                jet origin, $^\circ F$

T_m temperature of heat-transfer medium flowing over inside
                surface of windshield, $^\circ F$

θ_o temperature of air at surface-jet-nozzle exit above
                cockpit air temperature, $^\circ F$

θ_j maximum temperature of surface jet at any distance x
                from nozzle exit above cockpit air temperature, $^\circ F$

U_i over-all coefficient of heat transfer, inside surface
                of windshield to ambient air, Btu per hour, square
                foot, $^\circ F$

V_o air velocity at surface-jet-nozzle exit, feet per second

V_a velocity of airplane, feet per second

W heated air-flow rate out surface-jet-nozzle exit, pounds
                per second

x distance from jet origin to point under consideration
                $(x = L + e)$, feet

y thickness of windshield, feet

α angle of expansion of surface jet, degrees

ψ linear rate of change of $q_a + q_H$ with altitude, Btu
              per hour, square foot per foot

γ specific weight of air, pounds per cubic foot

ρ density of air, slugs per cubic foot

γ_w specific weight of windshield material, pounds per cubic
              foot

μ dynamic viscosity of air, pound-seconds per square foot

Φ $32.2 \, \mu l d_1^{\frac{1}{2}} \left(\dfrac{q_a + q_H}{0.16 \, k_a} \right)^{1.54}$

Subscripts

n   units of time

m   units of windshield thickness

ANALYSIS OF HEAT REQUIRED

In order to prevent the formation of fog on the inside surface
of a single-panel bullet-resisting windshield during diving flight,
it is necessary to maintain the temperature of this surface T_1
above the dew point of the adjacent cockpit air $T_{D.P.}$. A method
of calculating the heat required to provide the desired temperatures
of the inside surface of the windshield must be established before
consideration can be given to a means of supplying the required heat.

The state of temperature of a windshield at steady-state condi-
tions prior to dive flight may be expressed as

$$q_a = U_i \, (T_1 - T_a - \Delta T_k) = k_w \, (dT/dy) \qquad (1)$$

where

$$U_i = \frac{1}{\left(\dfrac{1}{h_o} \right) + \left(\dfrac{y}{k_w} \right)} \qquad (2)$$

and

$$\Delta t_k = \frac{c_k V_a^2}{2gJc_p} = T_k - T_a \qquad (3)$$

The equation for the external coefficient of heat transfer employed herein is

$$h_o = (3600)(0.036) \ c_p \ \gamma \ V_a \left(\frac{k_a}{3600\mu g c_p}\right)^{2/3} \left(\frac{\mu g}{L_w V_a \gamma}\right)^{1/5} \qquad (4)$$

This is the equation for the coefficient of heat transfer from an air stream to a flat surface given in reference 2 and, although it may not be strictly valid in the present application, it is considered sufficiently accurate as will be shown later.

As soon as the airplane on which a bullet-resisting windshield is installed enters diving flight, equation (1) is no longer valid; the flow of heat becomes transient. The ambient-air temperature T_a and density ρ as well as the coefficient of heat transfer h_o all vary with altitude. Due to the number of variables involved, it is impractical to treat the transient heat flow during diving flight by means of differential equations, and resort is made to the approximate method of E. Schmidt given in reference 3 for evaluating transient heat-flow conditions. In this method, the differential equations of transient heat flow are modified to permit a step-by-step solution in terms of finite quantities. The equations of reference 3 in the nomenclature of this report are listed below.

$$T_{m\Delta y, n\Delta t} = \frac{1}{2} \left[T_{(m-1)\Delta y, (n-1)\Delta t} + T_{(m+1)\Delta y, (n-1)\Delta t} \right] \qquad (5)$$

$$(T_1)_{o, n\Delta t} = \frac{(h_i)_{n\Delta t} \ \Delta y \ (T_m)_{n\Delta t} + k_w T_{\Delta y, n\Delta t}}{k_w + (h_i)_{n\Delta t} \ \Delta y} \qquad (6)$$

$$T_{(m+1)\Delta y, n\Delta t} = \frac{(h_o)_{n\Delta t} \, \Delta y \, (T_k)_{n\Delta t} + k_w T_{m\Delta y, n\Delta t}}{k_w + (h_o)_{n\Delta t} \, \Delta y} \qquad (7)$$

The units of time Δt and of windshield thickness Δy are related by the expression

$$\Delta t = \frac{\Delta y^2}{2a} \qquad (8)$$

The sketch of a windshield in figure 1 illustrates some of the quantities in the above equations.

The general method of solution of transient-heat-transfer problems using these equations is illustrated in reference 3. The calculations by the above method are long and tedious and must be made for particular values of all the variables. In order to facilitate the computations and also render them independent of the means by which the heat is transferred to the inside surface of the windshield, it is convenient to express equation (6) in terms of the heat flow through the inside surface of the windshield as

$$(T_1)_{o, n\Delta t} = \frac{(q_a + q_H)_{n\Delta t} \, \Delta y}{k_w} + T_{\Delta y, n\Delta t} \qquad (9)$$

where q_a is the heat input to the windshield at steady-state conditions prior to diving flight and $(q_a + q_H)_{n\Delta t}$ is the instantaneous heat input after $n\Delta t$ hours of diving flight.

For the purpose of calculations, it is necessary to postulate how $q_a + q_H$ will vary with altitude. Any variation may be chosen; however, herein it will be presumed that $q_a + q_H$ may be expressed as a linear function of altitude such that

$$\frac{d(q_a + q_H)}{dH} = \psi = \frac{q_H}{n\Delta h} \qquad (10)$$

where ΔH is the altitude traversed in the time Δt.

When equation (9) is employed in the step-by-step solution in place of equation (6), the temperature of the inside surface of the windshield results in terms of the heat input and may be expressed as

$$(T_1)_{o,n\Delta t} = (k_1)_{n\Delta t}\, q_a + (k_2)_{n\Delta t}\, \psi + (k_3)_{n\Delta t} \tag{11}$$

where k_1, k_2, and k_3 are functions of altitude and are dependent on the windshield material and thickness, the rate of descent, the airspeed and the altitude from which the diving flight is conducted.

Once the values of k_1, k_2, and k_3, have been established as functions of altitude for any particular installation and the dew point $T_{D.P.}$ of the air adjacent to the windshield at the critical altitude of fogging is known, a value of T_1 at that altitude must be chosen, such that T_1 is not less than $T_{D.P.}$ or

$$T_1 = T_a - (T_a - T_{D.P.}) + \Delta T \tag{12}$$

where ΔT is the temperature difference between the inside surface temperature of the windshield and the dew point of the air adjacent to the windshield at the critical altitude of windshield fogging. After establishing T_1 at the critical altitude of fogging, a value of ψ in equation (10) must be assumed and q_a at that altitude evaluated from equation (11). Then $q_a + q_H$ as a function of altitude is evident from equation (10) and T_1 as a function of altitude may be evaluated from equation (11). Many solutions are available depending on the value of ψ assumed. The value of ψ should be assumed such that the resultant values of $q_a + q_H$ may be correlated with the heat given up to the windshield by the heat-transfer medium employed. In the case of a heated-air, surface-jet, fog-prevention system the relationship

$$(q_a + q_H)_{n\Delta t} = [h_j(T_j - T_1)]_{n\Delta t} \tag{13}$$

must be satisfied.

RESULTS AND DISCUSSION

Heat Required

Using the relationships developed in the analysis, curves have
been established of k_1, k_2, and k_3 (equation (11)) as functions
of altitude for a $1\frac{1}{2}$-inch-thick, 2-foot long, bullet-resisting,
windshield with the following physical characteristics: average
specific weight γ_w of 168 pounds per cubic foot, average specific
heat c of 0.18 Btu per pound, °F and average conductance k/y of
3.91 Btu per hour, square foot, °F. The curves were established for
various constant rates of descent at constant true airspeeds of 200
and 400 miles per hour from pressure altitudes of 20,000 and 30,000
feet using the NACA standard air temperatures as T_k in equation (7)
and $\Delta y = \frac{1}{8} y$ in equation (8) for all calculations. The results of
these calculations are presented as figures 2 to 7 and a sample
calculation is illustrated in table I. Although these curves were
developed using standard air temperatures for T_k, they may be
applied equally well to other atmospheres wherein the temperature
lapse rate is the same as standard because the values of h_0
(equation (4)) employed in the solution vary only slightly with
temperature. Thus the heat flow $q_a + q_H$ may be calculated using
standard air temperatures and this heat flow will induce the same
temperature difference between the windshield inside-surface
temperature and the ambient-air temperature, or the same value of
$(T_a - T_{D.P.}) + \Delta T$ in equation (12) in any atmosphere in which a
standard temperature lapse rate prevails. The surface temperatures
resulting from this application of the curves will not be correct.
It follows from the above, however, that they may be corrected by
the relationship

$$(T_1-T_k)_{actual} = (T_1-T_k)_{standard\ air\ temperature\ for\ T_k} \qquad (14)$$

A study of figures 2 to 7 illustrates that the speed of the airplane
is not one of the most important factors in the solution when the
temperature T_1 is considered with respect to T_k. Solution of a
problem by use of the curves developed for 200 or 400 miles per hour
with other conditions the same will yield approximately the same
value of T_1 in equation (9). Thus it is evident that equation (4)
is sufficiently accurate for evaluating h_0.

It can be shown from the curves and equation (11) that for
cases where the rate of descent is high, over 5000 feet per minute,
little change is experienced in the values of T_i during diving
flight unless the value of ψ in equation (11) is large. For
instance, if the heat input were varied from q_a = 300 Btu per hour,
square foot, to $q_a + q_H$ = 200 Btu per hour, square foot, during
diving flight from 30,000 to 1,000 feet pressure altitude, respec-
tively, at a true airspeed of 400 miles per hour and a rate of
descent of 5000 feet per minute, the temperature of the inside
surface of the windshield T_i would decrease by only about 7° F.
In many installations, it is probable that the heat flow through
the windshield would increase with decreasing altitude, rather than
decrease, thus increasing the value of T_i by a small amount during
diving flight. This indicates that for many cases the design may be
carried out using steady-state heat-transfer considerations of the
heat flow through the inside surface of the windshield at the level
flight conditions from which the dive is initiated. The above
conclusion is based on constant airspeed before and during the dive,
which is not always the case. The conclusion is generally valid,
however, since it has been shown herein that airspeed has very little
effect on the values of T_i attained during diving flight. The
solution can thus be greatly simplified, necessitating the use of
equations (1), (2), (3), and (4) only. Therefore, in reality resort
need be made to the transient solution given in the analysis only
when ψ in equation (10) is negative. Resort should, however, be
made to the transient solution in order to conserve heat energy when
the rate of descent is low (less than 5000 feet per minute) or when
ψ has a large positive value.

Resort to the transient heat-flow solution is not necessary to
establish whether or not the steady-state solution is applicable in
any particular case. It follows from the previous discussion that
this may be accomplished by assuming T_i constant throughout diving
flight and calculating the heat flow to the inside surface of the
windshield. The values of $q_a + q_H$ thus evaluated may not be
accurate. However, if they indicate that q_H equals zero ($\psi = 0$),
then q_H is approximately zero. If they indicate that $q_a + q_H$
is larger than q_a (ψ is positive), the actual $q_a + q_H$ values
as determined by the transient solution will be larger than q_a but
not as large as the $q_a + q_H$ values indicated by the above computa-
tions in that the inside surface temperature of the windshield will
increase with decreasing altitude. If these approximate computations
indicate that $q_a + q_H$ is greater or equal to q_a throughout the
dive, the steady-state solution may be applied.

The effects of thickness and of conductivity of a bullet-resisting windshield on the values of k_1, k_2, and k_3 are illustrated in figure 8. The values of $k_1 q_a + k_3$ have been expressed in terms of temperature in this plot and the temperature for each curve at the altitude from which the diving flight is conducted has been made equivalent by using appropriate values of q_a. Without considering the k_2 values, the curves of $k_1 q_a + k_3$ as functions of altitude indicate the effects of windshield conductivity and thickness when the heat input is maintained constant throughout diving flight. Those curves show that increasing the thickness from $1\frac{1}{2}$ to 2 inches decreases the temperature change during diving flight, and decreasing the conductivity of the windshield material has a similar effect. The k_2 curves illustrate the effects of linearly changing the heat input with altitude for a change in windshield thickness and conductivity. This effect is just the opposite of that described for the $k_1 q_a + k_3$ curves. Also included on this figure are curves to show the effect on the solution of the number of layers into which the windshield is divided. Obviously the more layers chosen the more accurate the solution. Using eight layers, as was the case for all other computations of this report, the changes in the temperature of the windshield from step-to-step in the solution were uniform, and this is considered the criterion for appropriate use of the step-by-step calculation method employed.

The application of surface jets and electrical energy to provide the heat required is considered in the following discussion.

Application of Surface Jets

The properties of heated-surface jets are given in reference 1 and are listed below: (See surface-jet configuration, fig. 9.)

$$\frac{\theta_o}{\theta_j} = \left(\frac{c + 4d}{x}\right)^{-\frac{1}{2}} \tag{15}$$

$$\frac{h_j x}{k_a} = 0.16 \left[\frac{V_o x \rho}{\mu} \left(\frac{d}{d_1}\right)^{\frac{1}{2}}\right]^{0.65} \tag{16}$$

$$c = d/\tan \alpha \tag{17}$$

$$x = L + c \tag{18}$$

From these relationships, the heat transferred from a surface jet
to the inside surface of a single-panel bullet-resisting windshield
at any distance x from the jet origin may be expressed as

$$q_a + q_H = \frac{0.16 \, k_a}{x} \left[\frac{V_o x \rho}{\mu} \left(\frac{d}{d_1} \right)^{\frac{1}{2}} \right]^{0.65} (T_c + \theta_j - T_i) \tag{19}$$

or using the relationships

$$V_o = W/d l \gamma$$

$$\Phi = 32.2 \, \mu l d_1^{\frac{1}{2}} \left(\frac{q_a + q_H}{0.16 \, k_a} \right)^{1.54}$$

this expression may be written as

$$\frac{W(T_c + \theta_j - T_i)^{1.54}}{\Phi} = d^{0.5} x^{0.54} \tag{20}$$

Equation (20) suggests that, for a given value of Φ, T_c, T_i, θ_j,
and distance L, ($L = x - e$), there is an optimum nozzle depth
which will yield a minimum value of W. Evaluating dW/dd, W is a
minimum when

$$d = \left(\frac{1.54}{\frac{T_c - T_i}{\theta_j} + 1} - 1 \right) \frac{L \tan \alpha}{2.08} \tag{21}$$

The optimum jet temperature for a particular installation is
(equation (20)) the highest practical temperature which can be employed.
Two other important conclusions which can be drawn from the above rela-
tionships are: (1) equations (15) and (16) indicate that h_j and θ_j
decrease with increasing x, thus the design need be carried out only

for the point farthest distant from the jet nozzle exit at which fog
prevention is desired and (2) the cabin temperature T_c (equa-
tion (19)) should be maintained as high as practical in order to
minimize the temperature drop of the jet with distance x.

In the following, the correlations of these relationships with
those given for the heat required for fog prevention are discussed.

Steady-state solution.— The application of this solution requires
prior establishment of the variation of heat flow through the inside
surface of the windshield with altitude. This variation can be
approximated for the case of a surface jet by presuming T_i constant
and evaluating $q_a + q_H$ from equation (19) for various altitudes of
the dive. If these values are approximately equal to or larger than
q_a, the steady-state solution may be applied.

The steady-state solution is relatively simple. After the heat
required q_a is established ($q_H = 0$ in equation (19) for steady-
state considerations) from equation (1) and the highest windshield
inside-surface temperature for diving flight is known, a solution
of equation (20) will provide the surface-jet-nozzle depth required
for assumed values of W and $T_c + \theta_j - T_i$. In order that this
value of d will be an optimum, equation (21) must be satisfied.
The value of T_c used must be equivalent with that resulting from
cockpit heating considerations.

In order to simplify the procedure, a design chart which
includes plots of equations (20) and (21) is given in figure 10. The
use of this chart is illustrated in the solution of a sample problem
in Appendix A.

Transient solution.— Once the values of k_1, k_2, k_3,
$q_a + q_H$ and T_i as functions of altitude have been established for
a bullet-resisting windshield as set down in the analysis, the values
of $q_a + q_H$ must be correlated with the heat delivered to the inside
surface of the windshield by a heated surface jet by means of equa-
tion (13), together with equations (15), (16) and (17). These equa-
tions may be employed independently or grouped into the expression

$$q_a + q_H = 0.16\ k_a \left(\frac{\rho}{\mu d_1^{1/2}} \right)^{0.65} V_o^{0.65}\ d^{0.325}\ x^{-0.35} \left[T_c + \theta_o \left(\frac{4d + e}{x} \right)^{\frac{1}{2}} - T_i \right]$$

$$(22)$$

Thus it is necessary to decide upon values of V_o, d, T_c, and θ_o, such that equations (13) or (22) are satisfied throughout diving flight. The value of T_c must be commensurate with that resulting from cabin-heating considerations. The solution of a sample problem by the preceding method, the same problem solved for steady-state conditions in Appendix A, is provided in Appendix B.

Application of Electrical Heating

Recent developments have shown that transparent conductors of electricity can be applied to the surfaces of windshields. Thus some consideration of electrical heating by the conversion of electrical energy to heat energy at the inside surface of the windshield is merited in this report.

The heat balance at the inside surface of the windshield may be expressed as

$$q_a + q_H = q_E - q_i \tag{23}$$

where

$$q_i = h_c (T_i - T_c) \tag{24}$$

The quantity h_c may be expressed as

$$h_c = 0.27 (T_i - T_c)^{0.25} \tag{25}$$

if the windshield inside surface is vertical (reference 4). The coefficient 0.27 decreases to 0.20 as the attitude of the surface is changed from vertical to horizontal, with the surface under consideration facing downward. Since most windshields are placed at an angle of less than 90° to the horizontal, the use of 0.27 appears conservative. Using equation (25), equation (23) may be expressed as

$$q_a + q_H = q_E - 0.27 (T_i - T_c)^{1.25} \tag{26}$$

where

$$0.27 \, (T_i - T_c)^{1.25} = q_i \tag{27}$$

The quantity q_E will remain constant for any particular installation unless provisions are made to vary the electrical voltage supplied to the electrical heating system. If q_E is presumed constant, $q_a + q_H$ is dependent only on q_i. If, during diving flight, it is presumed that T_i maintains constant and that T_c maintains constant or increases, then q_i would decrease or remain constant. Thus it is evident that the design of fog-prevention systems employing the conversion of electrical energy to heat energy at the inside surface of the windshield may be carried out using the steady-state solution previously described.

It should be pointed out that the above considers only free convection and radiation of heat from the inside surface of the windshield. If the cockpit under consideration is drafty some forced convection may result and it may be of sufficient magnitude to require consideration.

CONCLUSIONS

The following conclusions on the heating of single-panel bullet-resisting windshields for fog prevention during diving flight are evident for the conditions analyzed.

1. During diving flight, when the rate of descent is high, over 5000 feet per minute, the inside-surface temperature of the windshield will remain approximately constant unless there is a large variation of heat flow through the inside surface of the windshield with altitude.

2. In designing fog-prevention systems for cases where the rate of descent is over 5000 feet per minute, the design may be carried out to provide the highest required inside-surface temperature of the windshield at the steady-state conditions prior to diving flight unless the heat flow through the inside windshield surface decreases appreciably with decreasing altitude.

3. Consideration of the transient heating of the windshield during diving flight is necessary to provide fog protection when the heat flow through the inside surface of the windshield decreases with decreasing altitude, and is desirable in order to conserve heat energy

when the rate of descent is low or the heat input to the windshield increases appreciably with decreasing altitude.

4. When employing heated surface jets for fog prevention it is advantageous to maintain the cockpit temperature as high as practical in order to reduce the rate of temperature decrease of the air jet with distance from the jet exit.

5. When employing heated surface jets for fog prevention, it is desirable to employ the highest practical jet temperatures in order to reduce the heat energy required.

6. If electrical energy is converted to heat energy at the inside surface of a bullet-resisting windshield and only free convection and radiation induce heat flow into the cockpit, the electrical heating system may be designed using steady-state heat-transfer considerations.

Ames Aeronautical Laboratory,
     National Advisory Committee for Aeronautics,
          Moffett Field, Calif., March 1947.

APPENDIX A

SAMPLE STEADY-STATE SOLUTION ON THE APPLICATION OF A HEATED
SURFACE JET FOR THE PREVENTION OF FOG ON THE INSIDE
SURFACE OF A SINGLE-PANEL BULLET-RESISTING
WINDSHIELD DURING DIVING FLIGHT.

The following data are considered as known:

Windshield

    Size: 12 in. wide by 24 in. long by 1-1/2 in. thick

    Conductance: 3.91 Btu per hr, ft^2, $^\circ$F

    Average specific weight: 168 lb per ft^3

Flight Conditions:

    Initial altitude: 30,000 ft (pressure)

    Initial ambient-air temperature: -30° F

    Ambient-air temperature lapse rate: Standard air

        lapse rate 0.003566° F per ft

    Rate of descent: 5000 ft per minute

    Airspeed, true, before and during diving flight:

        400 mph

Design Conditions:

    Altitude: 1000 ft (pressure)

    Ambient-air temperature : 73.5° F

    Protection required: Distance of 12 in. from jet-

        nozzle exit

In the steady-state solution, consideration of the transient

heat transfer during diving flight is neglected. It is presumed
that the initial surface temperature of the windshield will be
maintained throughout the dive. Thus the inside-surface temper-
ature of the windshield should be heated to 73.5° F prior to the
dive (in order to protect against vapor content up to 100-percent
relative humidity).

Solution

1. Heat required:

    External coefficient of heat transfer h_o from equation

    (4) = 30.5 Btu per hour, ft^2, ^oF

    Conductance of panel: given k_w/y = 3.91 Btu per hour,

    ft^2, ^oF

    Coefficient of heat transfer, inside surface of windshield to

    ambient air from equation (2), U_i = 3.45 Btu per hour

    ft^2, ^oF

    Heat required: from equation (1) q_a = 267 Btu per hour, ft

2. Evaluate $$\frac{W\left(T_c + \theta_j - T_i\right)^{1.54}}{\Phi}$$ in equation (20) and solve

    for d from figure 10

    From equation (20), assuming W = 0.1 pound per second and

    $(T_c + \theta_j - T_i)$ = 50° F, $W\dfrac{\left(T_c + \theta_j - T_i\right)^{1.54}}{\Phi}$ = 0.203

    From figure 10, d = 0.033 foot = 0.396 inch.

3. Establish T_c, θ_j, and T_o

    From figure 10, in order that the solution be an optimum

for the nozzle depth employed, $\dfrac{T_c - T_i}{T_j - T_c} = 0.065$

T_i (given) = 73.5° F, $T_j = T_i + (T_c + \theta_j - T_i) = 123.5°$ F

Thus

$$T_c = 76.5° \text{ F and } \theta_j = 47° \text{ F}$$

θ_o (from equation (15)) = 88° F and $T_o = \theta_o + T_c = 164.5°$ F

The value of T_c used must be commensurate with that result-ing from cabin-heating considerations.

APPENDIX B

SAMPLE TRANSIENT SOLUTION ON THE APPLICATION OF SURFACE JETS
FOR THE PREVENTION OF FOG ON THE INSIDE SURFACE OF A
BULLET-RESISTING WINDSHIELD DURING DIVING FLIGHT.

The known data for this solution are taken as the same as those given for the steady-state solution in Appendix A.

Solution

1. Heat required:

   k_1, k_2, and k_3 as a function of altitude.— Those data given in figures 3 and 6 are applicable using standard air temperatures as the kinetic air temperatures to calculate the heat required.

   Taking $\Delta T = 0$ in equation (12), $T_i = T_a = T_k - \dfrac{c_k V^2}{2gJc_p}$

   (T_i) 1000 ft = 30° F (for protection up to 100-percent relative humidity), assuming a value of c_k of 0.9.

2. $q_a + q_H$ as a function of altitude.— It was necessary to assume a value of ψ in order to calculate q_a (equation (11)). Herein values of ψ of 0, 0.0025, and -0.0025 were assumed to illustrate the effects of ψ on the solution. Values of

$q_a + q_H$ as functions of altitude were established from equation (10) and are tabulated in table II.

3. Windshield surface temperature.— The windshield surface temperatures were evaluated from equation (11). Since standard temperatures were employed in the above solution, the values of T_i were corrected by equation (14). The uncorrected and corrected values of T_i are tabulated in table II.

4. Application of surface jet.— Equations (13) or (22) must be satisfied. Thus assuming that $W = 0.1$ pound per second at 30,000 feet pressure altitude and varies such that V_j is constant ($V_j = 159$ ft/sec), $d = 0.033$ ft $= 0.396$ in., and $T_c = 70^\circ$ F, θ_0 was evaluated for each value of ψ as a function of altitude from use of equations (13), (15), (16), and (17), or equation (22). The values of T_0, $T_0 = \theta_0 + T_c$ for each value of ψ are tabulated in table II.

5. The value of T_c used in the calculations must be commensurate with that resulting from cabin—heating considerations.

REFERENCES

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2. Martinelli, R. C., Tribus, M., and Boelter, L. M. K: An Investigation of Aircraft Heaters. I — Elementary Heat Transfer Considerations in An Airplane. NACA ARR, Oct. 1942.

3. Schack, Alfred: Industrial Heat Transfer. Translated from the German by Hans Goldschmidt and Everett P. Partridge. John Wiley and Sons, Inc., 1933.

4. McAdams, William H.: Heat Transmission. Second Edition McGraw—Hill Book Co., Inc., 1942.

TABLE I.- CALCULATION OF k_1, k_2, AND k_3 FOR 3000 FEET PER MINUTE RATE OF DESCENT FROM 30,000 FEET ALTITUDE AT 200 MILES PER HOUR TRUE AIRSPEED, WINDSHIELD THICKNESS 1½ INCHES.

Time, Δt (hr)	Altitude, H (ft)	H_o (Btu/hr ft²)(°F) equation (4)	T_k (°F)	T_1 (°F) equation (9) $T_1=k_1\alpha+k_2\psi+k_3$			$(T)_{\Delta y}$ (°F) equation (5) $(T)_{\Delta y}=k_1\alpha+k_2\psi+k_3$			$(T)_{2\Delta y}$ (°F) equation (5) $(T)_{2\Delta y}=k_1\alpha+k_2\psi+k_3$			$(T)_{3\Delta y}$ (°F) equation (5) $(T)_{3\Delta y}=k_1\alpha+k_2\psi+k_3$			$(T)_{4\Delta y}$ (°F) equation (5) $(T)_{4\Delta y}=k_1\alpha+k_2\psi+k_3$			$(T)_{5\Delta y}$ (°F) equation (5) $(T)_{5\Delta y}=k_1\alpha+k_2\psi+k_3$			$(T)_{6\Delta y}$ (°F) equation (5) $(T)_{6\Delta y}=k_1\alpha+k_2\psi+k_3$			$(T)_{7\Delta y}$ (°F) equation (5) $(T)_{7\Delta y}=k_1\alpha+k_2\psi+k_3$			$(T)_{8\Delta y}$ (°F) equation (7) $(T)_{8\Delta y}=k_1\alpha+k_2\psi+k_3$		
				k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3
0Δt	30,000	0	-49.0	0.3128	0	-49.0	0.2808	0	-49.0	0.2488	0	-49.0	0.2168	0	-49.0	0.1848	0	-49.0	0.1528	0	-49.0	0.1208	0	-49.0	0.0888	0	-49.0	0.0568	0	-49.0
1Δt	28,688	18.5	-43.5	.3128	42	-49.0	.2808	0	-49.0	.2488	0	-49.0	.2168	0	-49.0	.1848	0	-49.0	.1528	0	-49.0	.1206	0	-49.0	.0888	0	-49.0	.0556	0	-47.0
2Δt	27,376	19.3	-38.5	.3128	105	-49.0	.2808	21	-49.0	.2488	0	-49.0	.2168	0	-49.0	.1848	0	-49.0	.1528	0	-49.0	.1203	0	-49.0	.0883	0	-48.0	.0547	0	-44.5
3Δt	26,064	20.2	-34.0	.3128	178	-49.0	.2808	52	-49.0	.2488	10	-49.0	.2168	0	-49.0	.1848	0	-49.0	.1523	0	-49.0	.1205	0	-48.5	.0857	0	-46.7	.0534	0	-41.7
4Δt	24,752	21.1	-29.0	.3128	261	-49.0	.2808	94	-49.0	.2488	26	-49.0	.2168	5	-49.0	.1846	0	-49.0	.1526	0	-48.7	.1202	0	-47.8	.0869	0	-45.1	.0520	0	-38.6
5Δt	23,440	22.0	-24.5	.3128	352	-49.0	.2808	143	-49.0	.2488	49	-49.0	.2168	13	-49.0	.1847	3	-48.9	.1525	0	-48.4	.1198	0	-46.9	.0861	0	-43.2	.0506	0	-35.5
6Δt	22,128	23.0	-20.0	.3128	451	-49.0	.2808	200	-49.0	.2488	78	-49.0	.2168	26	-49.0	.1847	7	-48.7	.1522	1	-47.9	.1193	0	-45.8	.0852	0	-41.2	.0491	0	-32.2
7Δt	20,816	24.0	-15.3	.3128	557	-49.0	.2808	264	-49.0	.2488	113	-49.0	.2168	42	-48.9	.1845	14	-48.4	.1520	3	-47.3	.1187	0	-44.5	.0842	0	-39.0	.0476	0	-28.8
8Δt	19,504	24.9	-10.5	.3128	669	-49.0	.2808	335	-49.0	.2488	153	-49.0	.2167	63	-48.7	.1844	22	-48.1	.1516	7	-46.4	.1181	1	-43.1	.0832	0	-36.7	.0464	0	-25.1
9Δt	18,192	25.9	-5.4	.3128	788	-49.0	.2808	411	-49.0	.2488	199	-48.8	.2166	87	-48.6	.1841	35	-47.5	.1513	12	-45.6	.1174	4	-41.5	.0823	0	-34.1	.0451	0	-21.2
10Δt	16,880	26.9	-1.2	.3128	911	-49.0	.2808	493	-49.0	.2487	249	-48.8	.2164	117	-48.2	.1839	50	-47.1	.1508	20	-44.5	.1168	6	-39.8	.0813	2	-31.3	.0437	0	-17.4
11Δt	15,568	27.9	3.6	.3127	1040	-48.9	.2807	580	-48.9	.2486	305	-48.6	.2163	149	-47.9	.1836	68	-46.3	.1503	28	-43.4	.1160	11	-37.9	.0802	3	-28.6	.0425	2	-13.3
12Δt	14,256	29.0	8.1	.3126	1176	-48.7	.2806	673	-48.7	.2485	364	-48.4	.2161	186	-47.4	.1833	89	-45.6	.1498	39	-42.1	.1152	16	-36.0	.0793	6	-25.6	.0413	3	-9.5
13Δt	12,944	30.0	13.0	.3125	1314	-46.5	.2805	770	-46.5	.2483	429	-48.0	.2159	226	-47.0	.1829	113	-44.7	.1493	53	-40.8	.1145	23	-33.9	.0782	9	-22.6	.0400	5	-5.2
14Δt	11,632	31.1	18.0	.3124	1457	-48.2	.2804	872	-48.2	.2482	496	-47.7	.2156	271	-46.3	.1825	139	-43.9	.1487	68	-39.3	.1138	31	-31.8	.0773	14	-19.5	.0388	7	-.6
15Δt	10,320	32.3	22.1	.3123	1606	-47.9	.2803	978	-47.9	.2480	571	-47.2	.2153	319	-45.8	.1821	169	-42.8	.1481	85	-37.9	.1130	41	-29.4	.0763	19	-16.3	.0376	9	3.1
16Δt	9,008	33.8	26.7	.3121	1757	-47.5	.2801	1089	-47.6	.2478	648	-46.8	.2150	370	-45.0	.1817	202	-41.9	.1475	105	-36.1	.1122	52	-27.1	.0753	25	-13.1	.0363	12	7.4
17Δt	7,696	34.9	31.7	.3120	1915	-47.2	.2800	1203	-47.2	.2476	730	-46.3	.2147	425	-44.3	.1812	237	-40.5	.1470	127	-34.5	.1114	65	-24.6	.0743	32	-9.9	.0352	15	9.6
18Δt	6,384	36.0	36.3	.3118	2075	-46.8	.2798	1322	-46.8	.2473	814	-45.6	.2144	483	-43.4	.1808	276	-39.4	.1463	152	-32.6	.1106	79	-22.2	.0733	40	-7.5	.0353	19	15.9
19Δt	5,072	37.3	40.8	.3115	2239	-46.3	.2795	1445	-46.3	.2471	902	-45.1	.2140	545	-42.6	.1803	317	-38.0	.1458	178	-30.8	.1098	96	-20.1	.0724	49	-3.2	.0331	22	20.6
20Δt	3,760	38.5	45.8	.3113	2406	-45.7	.2793	1570	-45.7	.2467	995	-44.5	.2137	609	-41.5	.1799	361	-36.7	.1451	206	-29.0	.1091	113	-17.0	.0714	59	+.3	.0321	27	25.4
21Δt	2,448	40.0	50.0	.3110	2578	-45.1	.2790	1700	-45.1	.2465	1089	-43.6	.2133	678	-40.6	.1794	407	-35.2	.1445	237	-26.9	.1083	132	-14.3	.0706	70	4.2	.0311	31	29.9
22Δt	1,136	41.5	55.0	.3107	2754	-44.3	.2787	1834	-44.3	.2461	1189	-42.8	.2129	748	-39.4	.1789	452	-33.7	.1439	269	-24.7	.1076	153	-10.3	.0697	81	7.8	.0300	35	34.7

TABLE II.— CALCULATED QUANTITIES OF $q_a + q_H$, T_1, AND T_o
REQUIRED FOR FOG PREVENTION IN SAMPLE TRANSIENT SOLUTION.

Calculated quantities	ψ	Pressure altitude (ft)			
		30,000	20,000	10,000	1,000
$q_a + q_H$ Btu per hour, square foot.	0	268	268	268	268
	.0025	250	275	300	322.5
	-.0025	291	266	241	218.5
T_1, °F using standard air temps. for T_k	0	29	29	29	30
	.0025	23.5	24.8	27.0	30
	-.0025	35	33.7	31.7	30
(T_1) corrected °F	0	73.0	73.0	73.0	73.5
	.0025	68.8	71.0	71.0	73.5
	-.0025	77.7	75.7	75.7	73.5
T_o, °F	0	162	141	126	117
	.0025	145	135	128	125
	-.0025	180.8	149	125	109

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$(T_i)_{0,n\Delta t}$

h_o

T_k

$T_{\Delta y, n\Delta t}$

$T_{m\Delta y, n\Delta t}$

$T_{2\Delta y, n\Delta t}$

$T_{(m+1)\Delta y, n\Delta t}$

$(q_a + q_h) \longrightarrow$

Inside surface

Outside surface
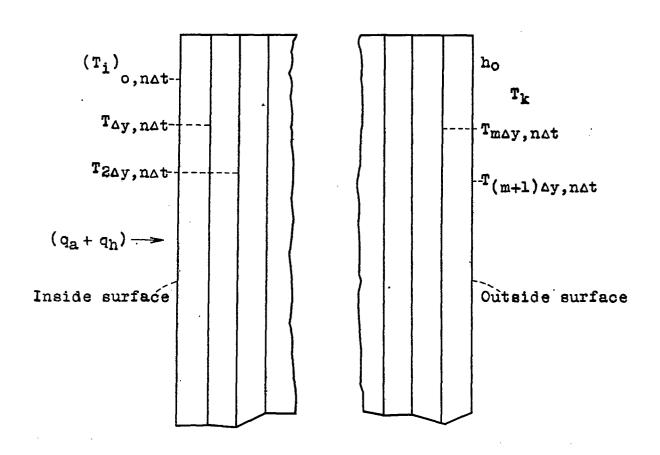
Figure 1.- Division of windshield for transient heat-flow
considerations.

Fig. 2
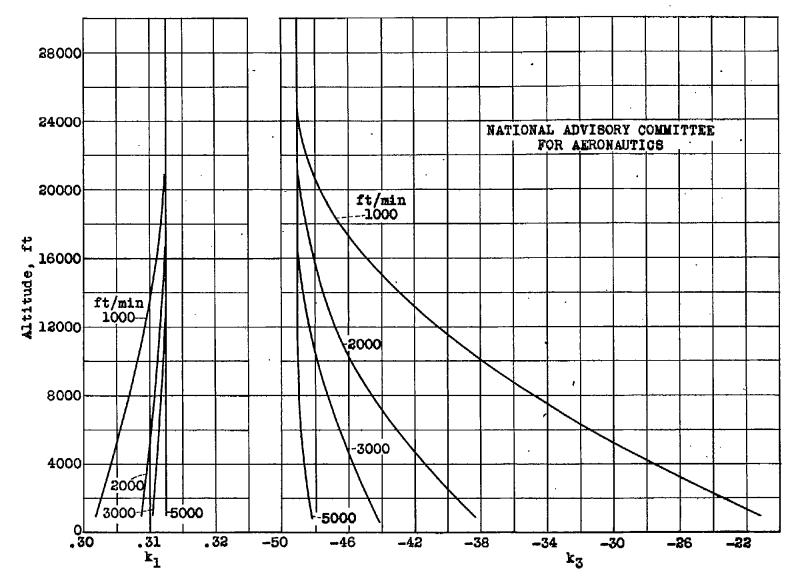
Figure 2.- Variation of k_1 and k_3 with altitude for various rates of descent from 30,000 feet altitude at 300 mph true airspeed. Windshield thickness 1-1/2 inches.

Fig. 3

Figure 3.— Variation of k_1 and k_3 with altitude for various rates of descent from 30,000 feet altitude at 400 mph true airspeed. Windshield thickness 1-1/2 inches.
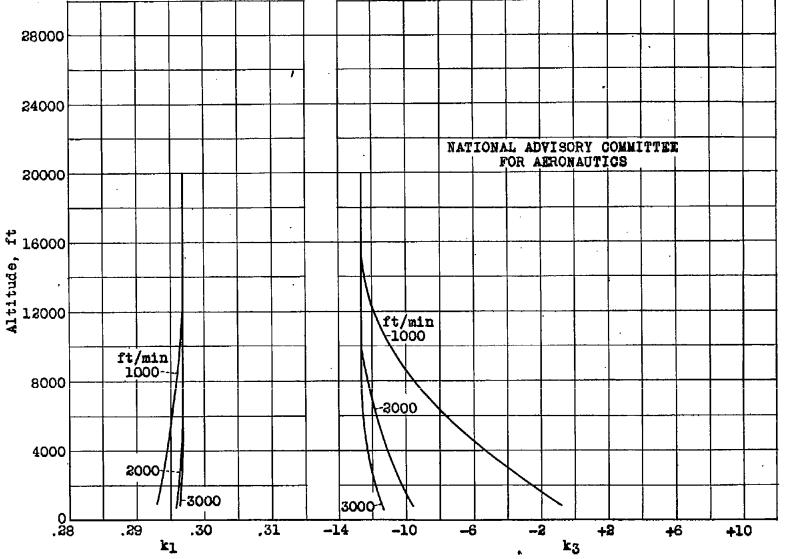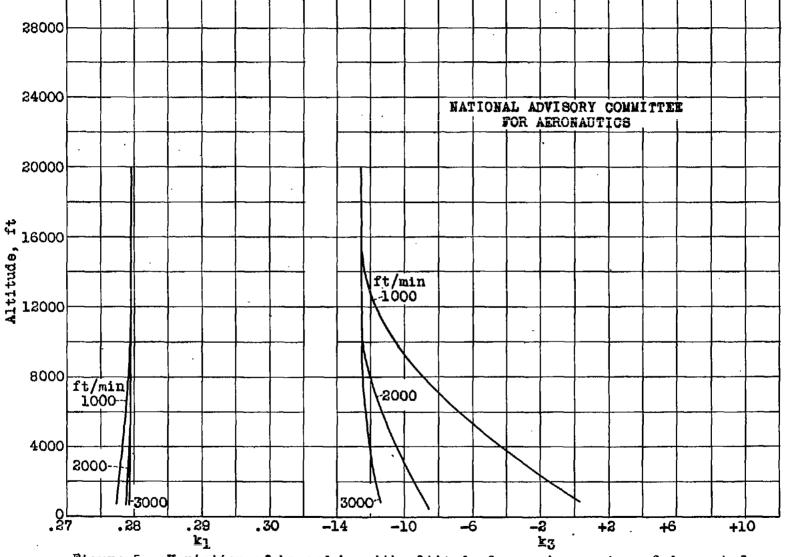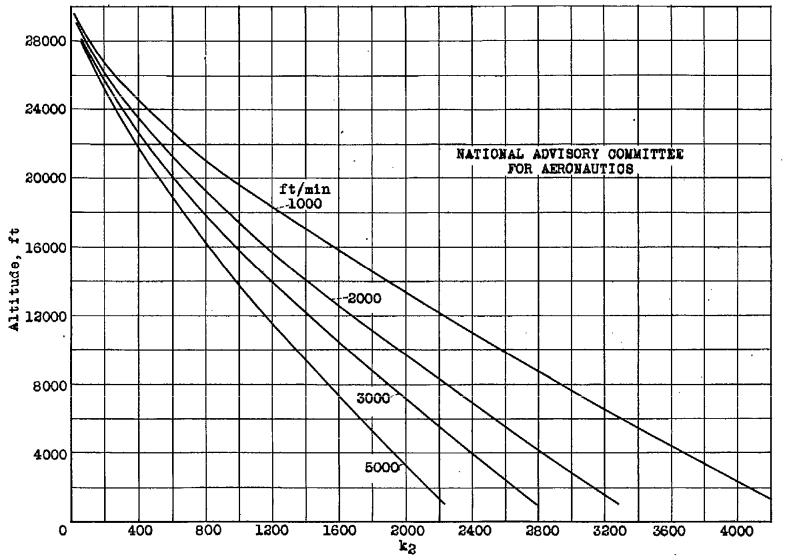
Fig. 4

Figure 4.- Variation of k_1 and k_3 with altitude for various rates of descent from 20,000 feet altitude at 200 mph true airspeed. Windshield thickness 1-1/2 inches.

Fig. 5

Figure 5.— Variation of k_1 and k_3 with altitude for various rates of descent from 20,000 feet altitude at 400 mph true airspeed. Windshield thickness 1-1/2 inches.

Fig. 6

Figure 6.— Variation of k_2 with altitude for various rates of descent from 30,000 feet altitude at 200 or 400 mph true airspeed. Windshield thickness 1-1/2 inches.
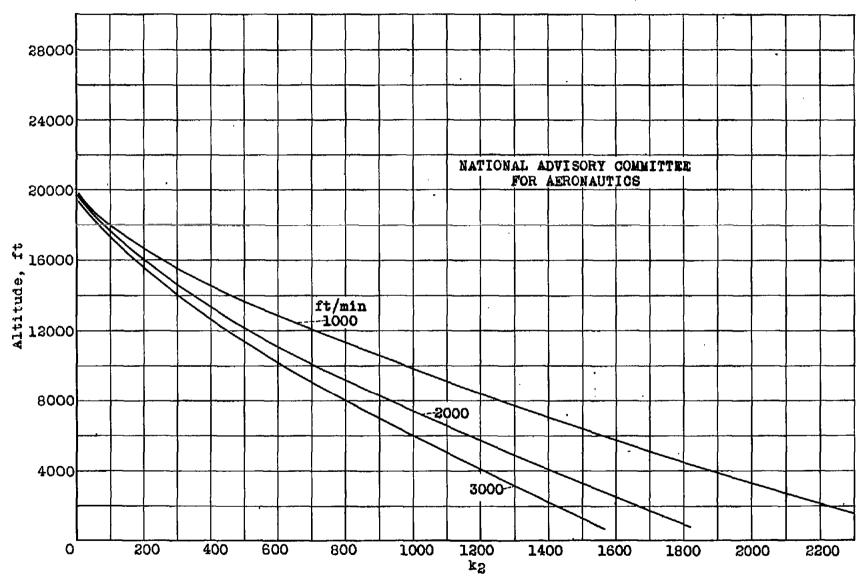
Figure 7.- Variation of k_2 with altitude for various rates of descent from 20,000 feet altitude at 200 or 400 mph true airspeed. Windshield thickness 1-1/2 inches.
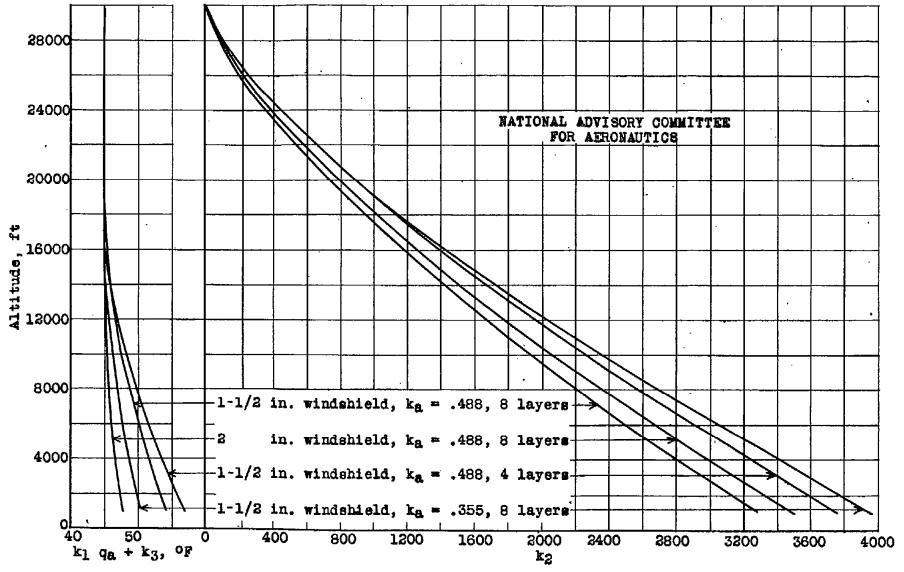
Fig. 7

Fig. 8

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Altitude, ft

1-1/2 in. windshield, k_a = .488, 8 layers

2 in. windshield, k_a = .488, 8 layers

1-1/2 in. windshield, k_a = .488, 4 layers

1-1/2 in. windshield, k_a = .355, 8 layers

$k_1 q_a + k_3$, °F

k_2

Figure 8.- Effect of windshield properties and number of windshield layers used in calculation of $k_1 q_a + k_3$ and k_2 curves for 2000 feet per minute descent from 30,000 feet at 200 mph true airspeed.
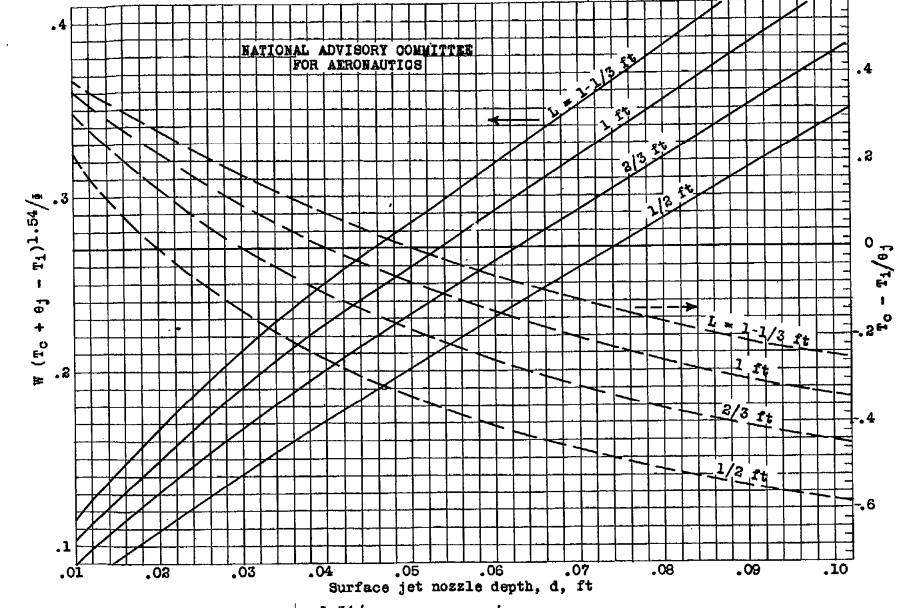
Figure 9.— Surface jet configuration.

Fig. 9

Fig. 10

Figure 10.— $W(T_c + \theta_j - T_i)^{1.54}/\Phi$ and $(T_c - T_i)/\theta_j$ as a function of surface-jet-nozzle depth, d.